An Improved Technique for PID Controller Tuning from Closed-Loop Tests

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The proportional-integral-derivative (PID) controller is the dominant type of controller used in current industrial practice. The behavior and capabilities of the PID controller are familiar to both the design engineer and the field operator, and it is relatively easy to tune manually by a variety of open-loop methods (Seborg et al., 1989). An important recent improvement in the PID controller tuning method is a self-tuning capability (Åström and Hägglund, 1984; Seborg et al., 1989), while the control loop is closed. Here we propose a new closed-loop tuning method for the PID controller

For closed-loop tuning of the PID controller, Yuwana and Seborg (YS, 1982) identified a first-order plus dead-time model from a few closed-loop transient data and calculated the controller settings by applying simple tuning rules such as the Ziegler-Nichols (ZN) method to the identified model. Åström and Hägglund (1984, 1988) used the limit cycles of relay feedback to find Nyquist points of the process and obtained the PID controller settings by applying the modified ZN method or a dominant pole design method. Nishikawa et al. (1984) used characteristic areas of the closed-loop transient response for process identification and a weighted integral square error method for calculation of the controller parameters. Krishnaswamy et al. (1987) identified frequency responses using a Fourier transform of closed-loop transient data.

The YS method can be applicable to most open-loop stable systems with dead time. The method was modified by Jutan and Rodriguez (1984), Lee (1989), and Chen (1989) for more robustness. However, since all of these methods use first-order plus dead-time models, the control performances for some processes are very poor. Here we improve the YS method by identifying processes with a second-order plus dead-time model under closed-loop conditions. We also employ a more elaborate frequency domain tuning method. To obtain a second-order plus dead-time model, a Taylor series expansion of the dead-time term is combined with the ultimate data matching technique of Chen (1989). To tune the PID controller, a frequency domain method based on methods of Edgar et al. (1981) and Harris and Mellichamp (1985) is applied, yielding controller settings much superior to the ZN method or the first-order methods.

Second-Order Plus Dead-Time Model Identification

To identify a model of a process, $G_p(s)$, a control system under proportional control $[G_c(s) = K_c]$ is tested. We choose a proportional controller with a sufficiently large gain so that the closed-loop system is underdamped. From a response curve to step change in set point of magnitude A, we can obtain an approximate model of the closed-loop system $G_{cl}(s)$ as (Yuwana and Seborg, 1982; Chen, 1989);

$$G_{cl}(s) = \frac{C(s)}{R(s)} = \frac{Ke^{-\gamma s}}{\tau^2 s^2 + 2\zeta\tau s + 1} \tag{1}$$

where

$$\begin{aligned}
K &= c_\infty / A \\
\zeta &= -\ln(\delta)/\sqrt{\pi^2 + \ln^2(\delta)} \\
\tau &= (t_{m1} - t_{p1})\sqrt{1 - \zeta^2}/\pi \\
\gamma &= 2t_{p1} - t_{m1} \\
c_\infty &= (c_{p1}c_{p2} - c_{m1}^2)/(c_{p1} + c_{p2} - 2c_{m1})
\end{aligned}$$

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$\delta = 1/2[(c_\infty - c_{m1})/(c_{p1} - c_\infty) + (c_{p2} - c_\infty)/(c_\infty - c_{m1})]$
c_{p1}, c_{p2} = 1st and 2nd peak values of the response, respectively
c_{m1} = 1st minimum value of the response
t_{p1}, t_{m1} = times corresponding to c_{p1} and c_{m1}, respectively

Since $G_{cl}(s) = K_c G_p(s)/[1 + K_c G_p(s)]$, we have

$$G_p(s) = \frac{G_{cl}(s)}{K_c[1 - G_{cl}(s)]} = \frac{Ke^{-\gamma s}}{K_c(\tau^2 s^2 + 2\zeta\tau s + 1 - Ke^{-\gamma s})} \quad (2)$$

From Eq. 2 we can obtain equations for the ultimate frequency and gain (ω_u, K_u)

$$\gamma\omega_u + \arctan[\{2\zeta\tau\omega_u + K\sin(\gamma\omega_u)\}/\{1 - \tau^2\omega_u^2 - K\cos(\gamma\omega_u)\}] = \pi \quad (3)$$

$$K_u = K_c \cdot \sqrt{[1 - \tau^2\omega_u^2 - K\cos(\gamma\omega_u)]^2 + [2\zeta\tau\omega_u + K\sin(\gamma\omega_u)]^2}/K$$

A second-order plus dead-time model can also be derived from Eq. 2 as

$$G_p(s) = \frac{(b_1 s + b_2)e^{-\theta s}}{s^2 + a_1 s + a_2} \quad (4)$$

where

$a_1 = (2\zeta\tau + K\gamma)/(\tau^2 - K\gamma^2/2)$
$a_2 = (1 - K)/(\tau^2 - K\gamma^2/2)$
$b_1 = \sqrt{(a_2 - \omega_u^2)^2 + (a_1\omega_u)^2 - (K_u b_2)^2}/(\omega_u K_u)$
$b_2 = a_2 K/[K_c(1 - K)]$
$\theta = [\pi + \arctan(b_1\omega_u/b_2) - \arctan\{a_1\omega_u/(a_2 - \omega_u^2)\}]/\omega_u$

In Eq. 4, a_1 and a_2 are obtained by the second-order Taylor series expansion of the dead-time term in the denominator of Eq. 2, b_2 is obtained from the steady state gains of Eqs. 2 and 4, and b_1 and θ are parameter values that give Eqs. 2 and 4 the same ultimate frequency and gain. The expressions for ultimate frequency and gain of Eq. 4 are the same as those of Chen (1989).

On-Line Tuning Method

Edgar et al. (1981) presented performance criteria to be used in interactive tuning of PID controllers in the frequency domain. They recommend PID controller settings such that the resonant peak amplitude ratio of the closed-loop system is not greater than 1.25 and the gain crossover frequency is as high as possible. Subsequently, Harris and Mellichamp (1985) automatically tuned PID controllers using an optimization technique with an index of performance IP as

$$IP = |M_p - 1.3|/0.13 + 0.5/\omega_p + |PM - 45|/45$$

where M_p and ω_p are the amplitude ratio and the angular frequency at resonant frequency, respectively, and PM is the phase margin. To simplify the calculation, they further used the ZN relation $\tau_i = 4\tau_d$ because it did not degrade the control performance and reduced computation time considerably. Both of these methods provide controller settings much superior to those of the ZN method. These methods, however, may require too much computer time, so for on-line calculations, we propose a simpler tuning method by approximation of the above methods.

The results of Harris and Mellichamp (1985) show that the closed-loop resonance peak value is near 1.3 and occurs near the Nyquist point with an angle of 225°. If we fix the peak value of the closed-loop resonance and let the peak occur at the Nyquist point of angle 225°, the tuning problem can be reformulated to search for the highest ω_p which satisfies the following constraints:

$$G_c(j\omega_p)G_p(j\omega_p) = re^{-j3\pi/4} \quad (5)$$

$$\omega_b \le 2\omega_p \quad (6)$$

where

$r = [1/\sqrt{2} - \sqrt{1/M_p^2 - 0.5}]/(1 - 1/M_p^2)$
$j = \sqrt{-1}$
ω_b = bandwidth of the closed-loop system

Constraint 6 is introduced so that the frequency cutoff rate is not too small (Chen, 1978); ω_b is the bandwidth frequency where the amplitude ratio $M = \sqrt{2}/2$.

The condition for resonance is

$$\frac{d}{d\omega}\{|G_c(j\omega)G_p(j\omega)/(1 + G_c(j\omega)G_p(j\omega))|\} = 0.$$

This, when combined with Eq. 5, gives the following relations for the PID settings.

$$K_c = r/\sqrt{[1 + q^2(\omega_p)][u^2(\omega_p) + v^2(\omega_p)]}$$

$$\tau_d = \frac{[\alpha\{u'(\omega_p) - v'(\omega_p)q(\omega_p) + v(\omega_p)q(\omega_p)/\omega_p\} + \beta\{v'(\omega_p) + u'(\omega_p)q(\omega_p) - u(\omega_p)q(\omega_p)/\omega_p\}]}{2[v(\omega_p) - u(w_p)]} \quad (7)$$

$$\tau_i = 1/[\omega_p\{\tau_d\omega_p - q(\omega_p)\}]$$

where

$$G_p(j\omega) = u(\omega) + jv(\omega)$$

$$u'(\omega) = \frac{du(\omega)}{d\omega}$$

$$v'(\omega) = \frac{dv(\omega)}{d\omega}$$

$$q(\omega) = \tan\left[\pi/4 - \arctan\{v(\omega)/u(\omega)\}\right]$$

$$\alpha = -\frac{r}{\sqrt{2}} - M_p^2\left(1 - \frac{r}{\sqrt{2}}\right)$$

$$\beta = (M_p^2 - 1)r/\sqrt{2}$$

Constraint 6 can be transformed to the following expression:

$$|G_c(j2\omega_p G_p(j2\omega_p)/\{1 + G_c(j2\omega_p)G_p(j2\omega_p)\}| \le \frac{\sqrt{2}}{2} \quad (8)$$

Therefore, we can solve the controller design problem as:

(i) Initialize $\omega_p = \omega_u/3$ and set M_p at one of two trial values (see discussion below)

(ii) Calculate the PID settings by Eq. 7 and the identified model in Eq. 4

(iii) Increase ω_p and repeat step (ii) until the constraint 8 is violated. The initial increment is $0.1\omega_u$ and the increment is doubled until the constraint is violated. Then it is successively halved until the error tolerance is satisfied. Typically 15 iterations are required to obtain an optimum value.

We have found that a low M_p value (near 1.02) results in better closed-loop responses for overdamped processes and a high M_p value (near 1.25) is better for underdamped processes. Thus, calculation is performed for two M_p values of 1.02 and 1.25, and settings which provide a higher value of ω_p are selected. For most of the problems solved, the optimal ω_p was found between $\omega_u/3$ and $3\omega_u$. A typical computation time is 0.2 s on an IBM 386/SX 16MHz personal computer.

Examples

We compare the proposed method with the previous methods of Yuwana and Seborg (1982) and Chen (1989) with respect to model accuracy and tuning performance. For these comparisons, three examples illustrating different processes have been selected that are the same ones studied by Yuwana and Seborg (1982):

(i) Overdamped process with dead time:

$$G_p(s) = \frac{e^{-3s}}{(s+1)^2(2s+1)}$$

(ii) Overdamped high-order process:

$$G_p(s) = \frac{1}{(s+1)^5}$$

(iii) Underdamped process with dead time:

$$G_p(s) = \frac{e^{-s}}{9s^2 + 2.4s + 1}$$

The integral of absolute error (IAE) between open-loop step responses of the model and the process was used to compare the identification accuracy of each tuning method. The integration time in IAE calculation is set to $3t_{p1}$ for processes (i) and (ii) and $5t_{p1}$ for process (iii). Table 1 shows models and IAE's. IAE values of models of the proposed LCE method are much lower

Table 1. Fitted Models of Three Test Processes and IAE for Different Closed-Loop Conditions

			Model Parameters					
Process	K_c	Method	a_1	a_2	b_1	b_2	θ	IAE
i	1.5	YS	0.255		0.255		4.69	1.60
		Chen	0.343		0.342		4.50	0.477
		LCE	0.754	0.173	0.261	0.172	4.19	0.137
	1.0	Chen	0.327		0.327		4.44	0.498
		LCE	0.730	0.173	0.213	0.173	3.97	0.149
	0.5	Chen	0.312		0.311		4.29	0.572
		LCE	0.747	0.184	0.161	0.183	3.65	0.178
ii	2.0	YS	0.244		0.244		3.18	2.17
		Chen	0.275		0.276		2.69	1.23
		LCE	0.759	0.208	0.116	0.209	1.99	0.258
	1.0	YS	0.286		0.286		3.00	1.48
		Chen	0.299		0.297		2.76	1.18
		LCE	0.878	0.246	0.128	0.245	2.15	0.289
iii	2.0	YS	0.213		0.213		4.20	5.89
		Chen	0.250		0.253		3.38	4.52
		LCE	0.265	0.109	0.027	0.110	1.25	0.334
	1.0	YS	0.257		0.257		4.90	5.99
		Chen	0.267		0.267		3.48	4.51
		LCE	0.280	0.113	0.024	0.113	1.32	0.292

Table 2. Tuning Results for Three Test Processes Using Three Closed-Loop Methods and Different Values of K_c

Method	Process	K_c	PID Controller				
			K_c	τ_i	τ_d	M_p	ω_b
YS-ZN	i	1.5	1.21	7.04	1.76	2.72	0.864
	ii	2.0	1.62	5.13	1.28	1.17	1.01
		1.0	1.57	4.76	1.19	1.14	0.967
	iii	2.0	1.47	6.62	1.65	1.34	0.790
		1.0	1.17	7.30	1.82	1.09	0.687
Chen-ZN	i	1.5	1.04	6.47	1.62	1.57	0.792
		1.0	1.08	6.45	1.61	1.72	0.802
		0.5	1.13	6.32	1.58	1.91	0.809
	ii	2.0	1.68	4.36	1.09	1.30	0.973
		1.0	1.56	4.41	1.10	1.17	0.936
	iii	2.0	1.51	5.38	1.34	1.72	0.750
		1.0	1.43	5.46	1.36	1.62	0.727
LCE	i	1.5	0.812	4.33	1.30	1.00	0.588
		1.0	0.855	4.33	1.43	1.02	0.640
		0.5	0.918	4.29	1.54	1.12	0.698
	ii	2.0	1.41	3.43	1.47	1.01	0.953
		1.0	1.31	3.45	1.33	1.01	0.859
	iii	2.0	1.77	4.14	2.74	1.23	1.34
		1.0	1.60	3.38	2.75	1.20	1.18

than those of models of YS and Chen [about one-third for problem (i), one-fourth for problem (ii), and one-tenth for problem (iii)]. IAE values for closed-loop step responses were also calculated and the same conclusions as above were obtained. Therefore, models obtained by LCE method approximate the processes more accurately than those of YS and Chen. As expected, much improvement is obtained for the underdamped process, which is difficult to approximate by the first-order plus dead-time model.

The tuning results are shown in Table 2. Peak amplitude ratio M_p and bandwidth ω_b are compared for the three methods and for different values of K_c. For the step set point change, a low peak amplitude ratio yields low overshoot and a high bandwidth causes a fast response time (Chen, 1978). In YS-ZN tuning and Chen-ZN tuning, the peak amplitude ratios are too high and fluctuate considerably for different values of K_c. The LCE method provides PID controller settings that yield moderate M_p

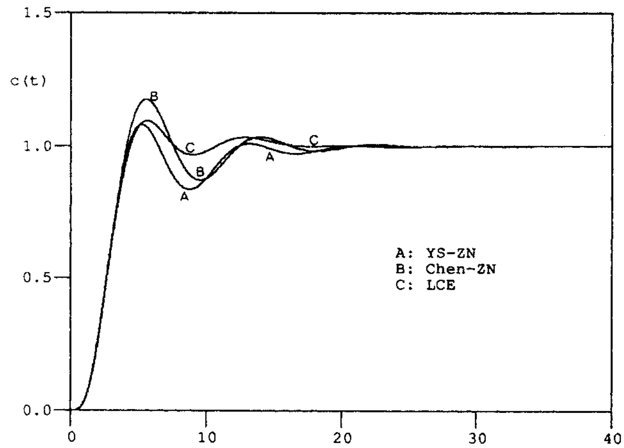

Figure 2. Closed-loop responses of PID control system for process (ii) tuned with the test run of $K_c = 2$.

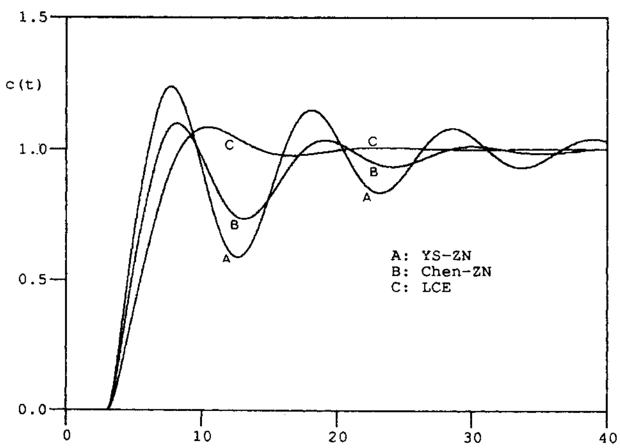

Figure 1. Closed-loop responses of PID control system for process (i) tuned with the test run of $K_c = 1.5$.

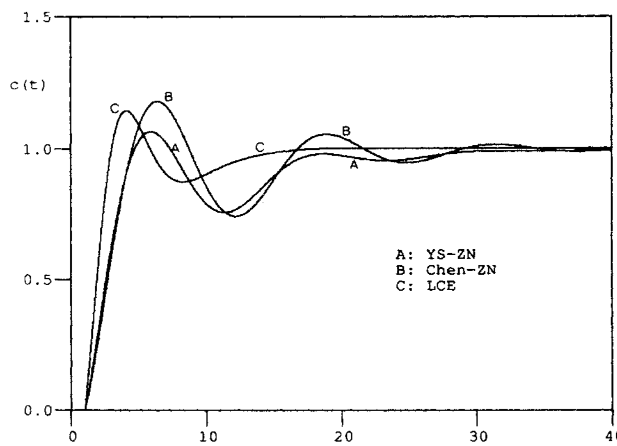

Figure 3. Closed-loop responses of PID control system for process (iii) tuned with the test run of $K_c = 2$.

and ω_b values consistently for all test problems and different values of K_c. Figures 1 to 3 show the improved tuning capability of the LCE method for the three test processes. We have also applied the LCE method to other processes such as an integrating process, an unstable process, and a process with inverse response, and obtained similar results.

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Notation

A = magnitude of set-point change
$c(t)$, $C(s)$ = controlled variable and its Laplace transform
c_∞, c_{m1}, c_{p1}, c_{p2} = steady-state value, first minimum, first and second peaks of c
$G_{cl}(s)$, $G_c(s)$, $G_p(s)$ = closed-loop, controller, and model or process transfer functions
K = closed-loop gain
K_c, K_p, K_u = PID controller, model and ultimate gain
M = amplitude ratio
M_p = amplitude ratio at resonance (peak amplitude ratio)
PM = phase margin of G_cG_p
P_u = ultimate period ($P_u = 2\pi/\omega_u$)
$r(t)$, $R(s)$ = set point variable and its Laplace transform
s = Laplace variable
t_{m1}, t_{p1} = time of first minimum and first peak
$u(\omega)$, $v(\omega)$ = real and imaginary part of $G_p(j\omega)$

Greek letters

δ = overshoot
γ = dead time of closed-loop response
θ = dead time of process model
τ = time constant of closed-loop transfer function
τ_i, τ_d = integral and derivative time of PID controller
ω, ω_b, ω_u, ω_p = angular frequency, bandwidth, ultimate and resonance angular frequency
ζ = damping coefficient for second-order model

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